

Lingua Project

(5) The denotations of expressions

(Sec. 6.1 – 6.4)

The book "**Denotational Engineering**" may be downloaded from:
<https://moznainaczej.com.pl/what-has-been-done/the-book>

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A recapitulation of former lecture

Classes

cla	: Class	= Identifier x TypEnv x MetEnv x Objecton	classes
tye	: TypEnv	= Identifier \Rightarrow Type $\{\Theta\}$	type environments
mee	: MetEnv	= Identifier \Rightarrow Method	method environments
met	: Methods	= ProSig PrePro	methods

A recapitulation of former lecture

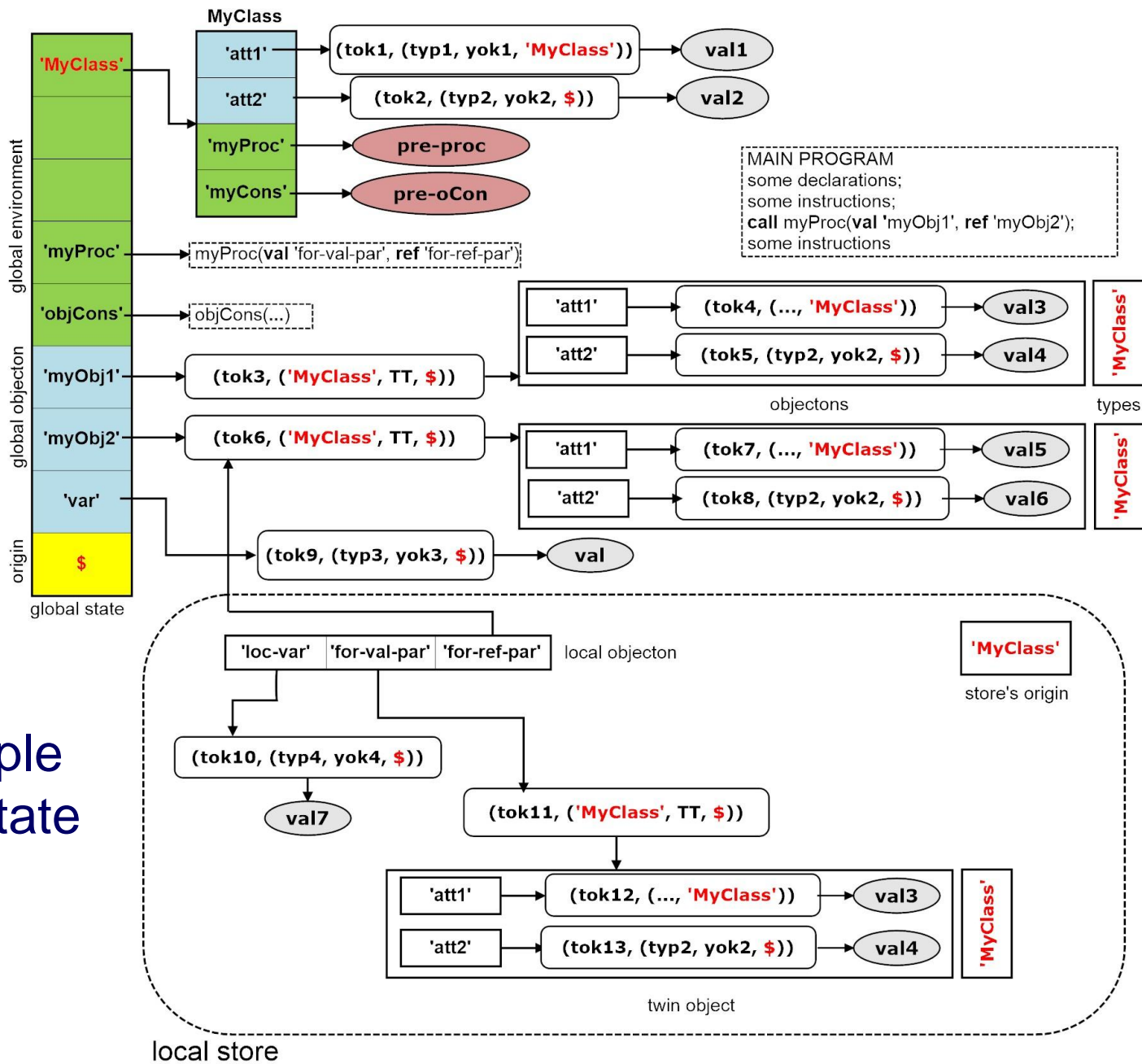
Stores and states

sta	: State	= Env x Store	states
env	: Env	= ClaEnv x ProEnv x CovRel	environments
cle	: ClaEnv	= Identifier \Rightarrow Class	class environments
pre	: ProEnv	= ProInd \Rightarrow Procedure	procedure environments
pri	: ProInd	= Identifier x Identifier	procedure indicators
sto	: Store	= Objecton x Deposit x OriTag x SetFreTok x (Error {'OK'})	stores
cov	: CovRel	= Sub.((DatTyp x DatTyp) (ObjTyp x ObjTyp))	covering relations
sft	: SetFreTok	= Set.Token	sets of (free) tokens
obn	: Objecton	= Identifier \Rightarrow Reference	
dep	: Deposit	= Reference \Rightarrow Value	

{ \$ } | Identifier

get-tok : SetFeeTok \rightarrow {tok} | SetFeeTok

An example of a state



Selected carriers of the algebra of denotations

Primitive carriers

$\text{ide} : \text{Identifier}$	$= \dots$	identifiers
$\text{prs} : \text{PriSta}$	$= \{\text{'private'}, \text{'public'}\}$	privacy statuses indicators
$\text{loi} : \text{ListOfIde}$	$= \text{Identifier}^{\text{c}*}$	lists of identifiers
$\text{cli} : \text{ClInd}$	$= \{\text{'empty-class'}\} \mid \text{Identifier}$	class indicators

Applicative carriers

$\text{ved} : \text{ValExpDen} = \text{WfState}$	$\rightarrow \text{ValueE}$	value-expression denotations
$\text{yok} : \text{YokExpDen} = \text{WfState}$	$\rightarrow \text{YokeE}$	yoke-expression denotations
$\text{ted} : \text{TypExpDen} = \text{WfState}$	$\mapsto \text{TypeE}$	type-expression denotations
$\text{red} : \text{RefExpDen} = \text{WfState}$	$\mapsto \text{ReferenceE}$	reference-expression den.

Imperative carriers

$\text{dcd} : \text{DecDen} = \text{WfState}$	$\rightarrow \text{WfState}$	declaration denotations
$\text{ind} : \text{InsDen} = \text{WfState}$	$\rightarrow \text{WfState}$	instruction denotations
$\text{ppd} : \text{ProPreDen} = \text{WfState}$	$\rightarrow \text{WfState}$	program-preamble den.
$\text{prd} : \text{ProDen} = \text{WfState}$	$\rightarrow \text{WfState}$	program denotations

Selected constructors of value-expression denotations

Constructors of fixed-value-expression denotations

ved-bo.boo	:		\mapsto ValExpDen	for boo : {tt, ff}
ved-in.int	:		\mapsto ValExpDen	for int : Integer
ved-re.rea	:		\mapsto ValExpDen	for rea : Real
ved-tx.tex	:		\mapsto ValExpDen	for tex : Text

Constructors of selection-expression denotations

ved-variable	:	Identifier	\mapsto ValExpDen
ved-attribute	:	Identifier x Identifier	\mapsto ValExpDen

Constructor of functional procedure calls

ved-call-fun-pro : Identifier x Identifier x ActParDen \mapsto ValExpDen

...

Selected constructors of value-expression denotations

Constructors based on typed-data constructors (examples)

ved-divide-re	: ValExpDen x ValExpDen	\mapsto ValExpDen
ved-create-li	: ValExpDen	\mapsto ValExpDen
ved-get-from-rc	: ValExpDen x Identifier	\mapsto ValExpDen
ved-get-from-ar	: ValExpDen x ValExpDen	\mapsto ValExpDen

Constructors of boolean-expression denotations

equal	: ValExpDen x ValExpDen	\mapsto ValExpDen
less	: ValExpDen x ValExpDen	\mapsto ValExpDen
ved-and	: ValExpDen x ValExpDen	\mapsto ValExpDen
ved-or	: ValExpDen x ValExpDen	\mapsto ValExpDen
ved-not	: ValExpDen	\mapsto ValExpDen

Conditional-expression constructor

ved-if	: ValExpDen x ValExpDen x ValExpDen	\mapsto ValExpDen
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Selected definitions of constructors of value-expression denotations

A constant-value expression

$\text{ved-int.3} : \mapsto \text{ValExpDen}$ i.e.

$\text{ved-int.3} : \mapsto \text{WfState} \rightarrow \text{Value} \mid \text{Error}$

$\text{ved-int.3}().\text{sta} =$

$\text{is-error.sta} \rightarrow \text{error.sta}$

true $\rightarrow (3, \text{'integer'})$

A single-variable expression

$\text{ved-variable} : \text{Identifier} \mapsto \text{ValExpDen}$ i.e.

$\text{ved-variable} : \text{Identifier} \mapsto \text{WfState} \rightarrow \text{ValueE}$

$\text{ved-variable}.\text{ide}.\text{sta}$

$\text{is-error.sta} \rightarrow \text{error.sta}$

let

$(\text{env}, (\text{obn}, \text{dep}, \text{st-ota}, \text{sft}, \text{'OK'})) = \text{sta}$

$\text{obn}.\text{ide} = ? \rightarrow \text{'variable not declared'}$

$\text{dep}.\text{(obn.ide)} = ? \rightarrow \text{'variable not initialized'}$

true $\rightarrow \text{dep}.\text{(obn.ide)}$

Selected definitions of constructors of value-expression denotations

An object-attribute expression

$\text{ved-attribute} : \text{ValExpDen} \times \text{Identifier} \mapsto \text{ValExpDen}$

i.e.

$\text{ved-attribute} : \text{ValExpDen} \times \text{Identifier} \mapsto \text{WfState} \rightarrow \text{ValueE}$

$\text{ved-attribute}.\text{(ved, at-ide)}.sta =$

$\text{is-error}.sta \rightarrow \text{error}.sta$

$\text{ved}.sta = ? \rightarrow ?$

$\text{ved}.sta : \text{Error} \rightarrow \text{ved}.sta$

$\text{ved}.sta /: \text{Object} \rightarrow \text{'object expected'}$

let

$(\text{ob-obn}, \text{cl-ide}) = \text{ved}.sta$

$\text{obn}.\text{at-ide} = ? \rightarrow \text{'attribute unknown'}$

let

$(\text{tok}, (\text{typ}, \text{yok}, \text{ota})) = \text{obn}.\text{ide}$ the reference of ide in obn

$(\text{env}, (\text{obn}, \text{dep}, \text{st-ota}, \text{sft}, \text{'OK'})) = \text{sta}$

$\text{dep}.\text{(obn.at-ide)} = ? \rightarrow \text{'attribute not initialized'}$

$\text{ota} \neq \$ \text{ and } \text{ota} \neq \text{st-ota} \rightarrow \text{'attribute not visible'}$

true $\rightarrow \text{dep}.\text{(ob-obn.at-ide)}$

Selected definitions of constructors of value-expression denotations

Division of real numbers

ved-divide-re: ValExpDen x ValExpDen \mapsto ValExpDen i.e.

ved-divide-re: ValExpDen x ValExpDen \mapsto WfState \rightarrow ValueE

ved-divide-re.(ved-1, ved-2).sta =

is-error.sta \rightarrow error.sta

ved-i.sta = ? \rightarrow ? for i = 1,2

ved-i.sta : Error \rightarrow ved-i.sta for i = 1,2

let

val-i = ved-i.sta for i = 1,2

val = td-divide-re.(val-1, val-2)

true \rightarrow val

Selected definitions of constructors of value-expression denotations

The equality of values (a boolean expression)

$\text{equal} : \text{ValExpDen} \times \text{ValExpDen} \mapsto \text{ValExpDen}$ i.e.

$\text{equal} : \text{ValExpDen} \times \text{ValExpDen} \mapsto \text{WfState} \rightarrow \text{Value} \mid \text{Error}$

$\text{equal}.\text{(ved-1, ved-2).sta} =$

$\text{is-error.sta} \rightarrow \text{error.sta}$

$\text{ved-i.sta} = ? \rightarrow ?$ for $i = 1,2$

$\text{ved-i.sta} : \text{Error} \rightarrow \text{ved-i.sta}$ for $i = 1,2$

let

$(\text{cor-i, typ-i}) = \text{ved-i.sta}$ for $i = 1,2$

$\text{typ-1} \neq \text{typ-2} \rightarrow$ 'compared values must be of the same type'

not $\text{comparable.typ-1} \rightarrow$ 'values not comparable'

$\text{cor-1} = \text{cor-2} \rightarrow$ (tt, 'boolean')

true \rightarrow (ff, 'boolean')

Selected constructors of type-expression denotations

ted-create-bo :		\mapsto TypExpDen
ted-create-in :		\mapsto TypExpDen
ted-create-re :		\mapsto TypExpDen
ted-create-tx :		\mapsto TypExpDen
ted-create-ot :	Identifier	\mapsto TypExpDen
ted-constant :	Identifier x Identifier	\mapsto TypExpDen
ted-create-li :	TypExpDen	\mapsto TypExpDen
ted-create-ar :	TypExpDen	\mapsto TypExpDen
ted-create-re :	Identifier x TypExpDen	\mapsto TypExpDen
ted-put-to-re :	Identifier x TypExpDen x TypExpDen	\mapsto TypExpDen

Selected definitions of constructors of type-expression denotations

Boolean-type expression denotation

```
ted-create-bo.().sta =  
  is-error.sta → error.sta  
  true       → 'boolean'
```

Object-type expression denotation

```
ted-create-ot.ide.sta =  
  is-error.sta → error.sta  
  true       → ide
```

One-attribute-record-type expression denotation

```
ted-create-re.(ide, ted).sta =  
  is-error.sta → error.sta  
  ted.sta : Error → ted.sta  
  let  
    typ = ted.sta  
  true       → ty-create-re.(ide, typ): (R', [ide/typ])
```

Selected definitions of constructors of type-expression denotations

Type-constant expression denotation

ted-constant.(ide-cl, ide-ty).sta =

is-error.sta \rightarrow error.sta

let

((cle, mee, cov), sto) = sta

cle.ide-cl = ? \rightarrow 'class unknown'

let

(ide-cl, tye, mee, obn) = cle.ide-cl

tye.ide-ty = ? \rightarrow 'type unknown'

tye.ide-ty = Θ \rightarrow 'type not concretized'

true \rightarrow tye.ide-ty

well-formedness of sta

Two constructors of reference-expression denotations

A variable reference

ref-variable : Identifier \mapsto RefExpDen

An attribute reference

ref-attribute : ValExpDen x Identifier \mapsto RefExpDen

The definition of a constructor of reference-expression denotations

ref-variable : Identifier \mapsto RefExpDen i.e.
ref-variable : Identifier \mapsto WfState \mapsto ReferenceE
ref-variable.ide.sta =
 is-error.sta \rightarrow error.sta
 let
 (env, (obn, dep, st-ota, sft, 'OK')) = sta
 obn.ide = ? \rightarrow 'variable not declared'
 true \rightarrow obn.ide

The definition of a constructor of reference-expression denotations

ref-attribute : ValExpDen x Identifier \mapsto RefExpDen i.e.

ref-attribute : ValExpDen x Identifier \mapsto WfState \mapsto ReferenceE

ref-attribute.(ved, at-ide).sta =

is-error.sta \rightarrow error.sta

ved.sta = ? \rightarrow ?

ved.sta : Error \rightarrow ved.sta

ved.sta /: Object \rightarrow 'object expected'

let

(va-obn, va-ide) = ved.sta

(env, (obn, dep, st-ota, sft, 'OK')) = sta

va-obn.at-ide = ? \rightarrow 'attribute not declared'

let

(tok, (typ, yok, at-ota)) = va-obn.at-ide

at-ota \neq \$ **and** at-ota \neq st-ota \rightarrow 'attribute not visible'

true \rightarrow va-obn.at-ide



Thank you for
your attention